

Titel: Cutting planes for binary optimal control problems

Abstract:

Optimal control problems containing both integrality and partial differential equation (PDE) constraints are very challenging in practice. The most wide-spread solution approach is to directly discretize the entire problem, resulting in huge nonlinear mixed-integer optimization problems that can be solved to proven optimality only in very small dimensions. In this talk, we propose an outer approximation approach to efficiently solve such problems in the case of certain semilinear elliptic PDEs with static integer controls over arbitrary combinatorial structures. The basic idea is to decompose the problem into an integer linear programming master problem and a subproblem for calculating linear cutting planes. These cutting planes rely on the pointwise concavity of the PDE solution operator in terms of the control variables, which we prove in the case of PDEs with a non-decreasing convex nonlinear part. In particular, the projection of the feasible region to the control space (after relaxing integrality) turns out to be convex, while the continuous relaxations obtained in the direct discretization approach are non-convex in general.